

Applying Quantitative Concepts

STUDENT ID:

[Pick the date]

Sample Assignment

Question 1

Getting Started

The problem essentially aims to determine the time taken for a concerned person to travel from Earth to Europa based on the distance between the two and the speed to the space shuttle. Using the relation between distance, time and speed, given the data corresponding to distance and speed, the time can be determined. However, before that the distance would need to be determined taking the speed of light and time data given. The second part involves determining the maximum age of the colonists on departure from earth. This would essentially involve the ageing process expected from the time taken to travel from Europa to Earth. Also, since the maximum age is being asked hence, the larger of the two distances would be considered so that the ageing process leads to the maximum value. Further, certain assumptions would also need to be considered in order to determine the maximum age.

Calculations

Speed of light = 3×10^8 m/s

Minimum time taken by light to cover distance between Earth and Europa = 33 minutes or
(33*60) = 1980 seconds

Maximum time taken by light to cover distance between Earth and Europa = 54 minutes or
(54*60) = 3240 seconds

Distance = Speed * Time

Hence, minimum distance between Earth and Europa = $3 \times 10^8 \times 1980 = 5.94 \times 10^{11}$ m/s

Also, maximum distance between Earth and Europa = $3 \times 10^8 \times 3240 = 9.72 \times 10^{11}$ m/s

Average distance between Earth and Europa = (Minimum Distance + Maximum Distance)/2
= $[(5.94 + 9.72)/2] \times 10^{11}$ m/s = 8.33×10^{11} m/s

Speed of the space shuttle = 9200 m/s

Hence, time taken to travel to Europa = $(8.33 \times 10^{11} / 9200) = 90,543,478$ seconds or 25,150.97 hours or 2.87 years

In order to find the maximum age, it is essential to consider the maximum distance and the time corresponding to the same.

Time using maximum distance = $(9.72 \times 10^{11} / 9200) = 29,347.83$ hours or 3.35 years

Conclusion

Based on the above computations, it is apparent that on an average it would take 2.87 years to travel from Earth to Europa assuming that the speed of the space shuttle remains the same as highlighted in the question. This essentially would highlight the project duration and supplies that would be required for such an exploration mission. Further, the time taken to arrive back at the earth also needs to be added. The time may be marginally reduced by ensuring that the altering the trajectory to ensure that the distance travelled is equal to the minimum distance between the two.

For any given colonist that comes to comes would at maximum be older by 3.35 when he/she leaves the earth assuming that the time spent on earth is zero. The precise maximum age would depend on the age of the colonist at the beginning of the earth mission. Also, it is assumed that the technology that the colonists have would be comparable to humans and hence the speed of their space shuttles would be comparable to ours. A better technology may lead to lower time as the speed of the space shuttle could potentially be higher.

An alternative method of computation of the maximum age could have been calculating the age assuming the speed of light and hence making adjustments by comparing the speed of light and space shuttle. However, it is not a preferable approach here and hence not used for computing.

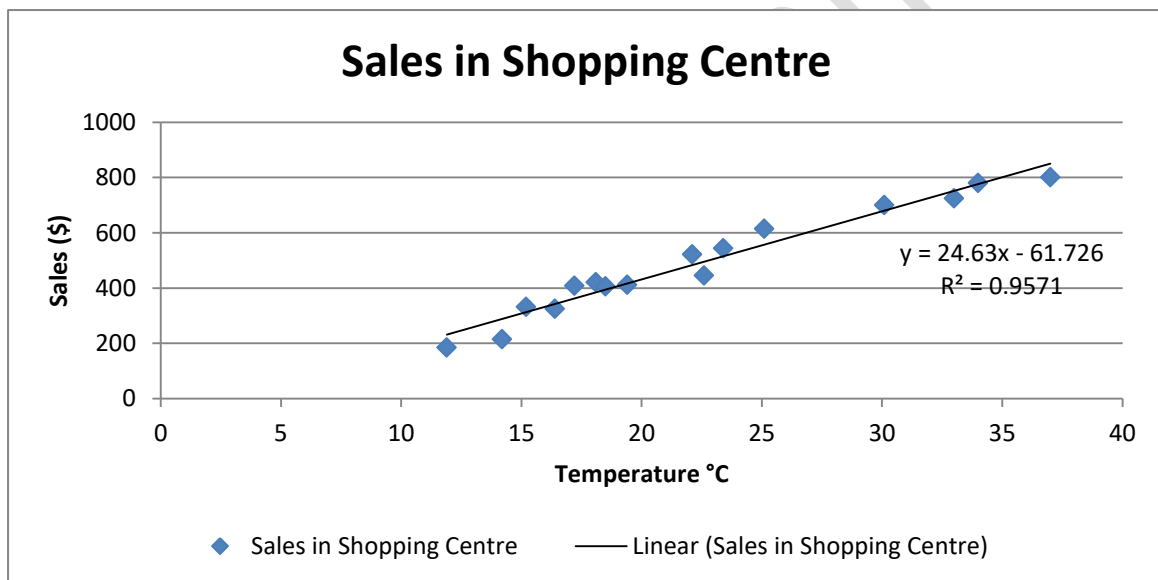
Question 2

Getting Started

The objective of the given problem is to determine the relationship between temperature and ice cream sales. For this, two different ice cream sellers have been chosen so as to understand as to whose sales is more dependent on temperature. The underlying analysis that would be used for determining the relationship between the given variables would be linear regression. The slope of the regression equation would determine the precise impact of temperature. Also, a scatter plot would also be drawn based on the given data which would indicate the nature of relationship between temperature and ice cream sales.

Calculation

Case 1: Sale at the shopping centre



The calculation of the regression equation is indicated below.

Temperature (X)	Sales in Shopping Centre (Y)	X ²	Y ²	XY
11.9	185	141.61	34225	2201.5
14.2	215	201.64	46225	3053
15.2	332	231.04	110224	5046.4
16.4	325	268.96	105625	5330
17.2	408	295.84	166464	7017.6
18.1	421	327.61	177241	7620.1
18.5	406	342.25	164836	7511
19.4	412	376.36	169744	7992.8
22.1	522	488.41	272484	11536.2
22.6	445	510.76	198025	10057
23.4	544	547.56	295936	12729.6
25.1	614	630.01	376996	15411.4
30.1	700	906.01	490000	21070
33	725	1089	525625	23925
34	780	1156	608400	26520
37	801	1369	641601	29637

$$\sum X = 358.2, \sum Y = 7835, \sum X^2 = 8882.06, \sum Y^2 = 4383651, \sum XY = 196658.6, n=16$$

The relevant formula for the coefficients of the regression equation is as follows.

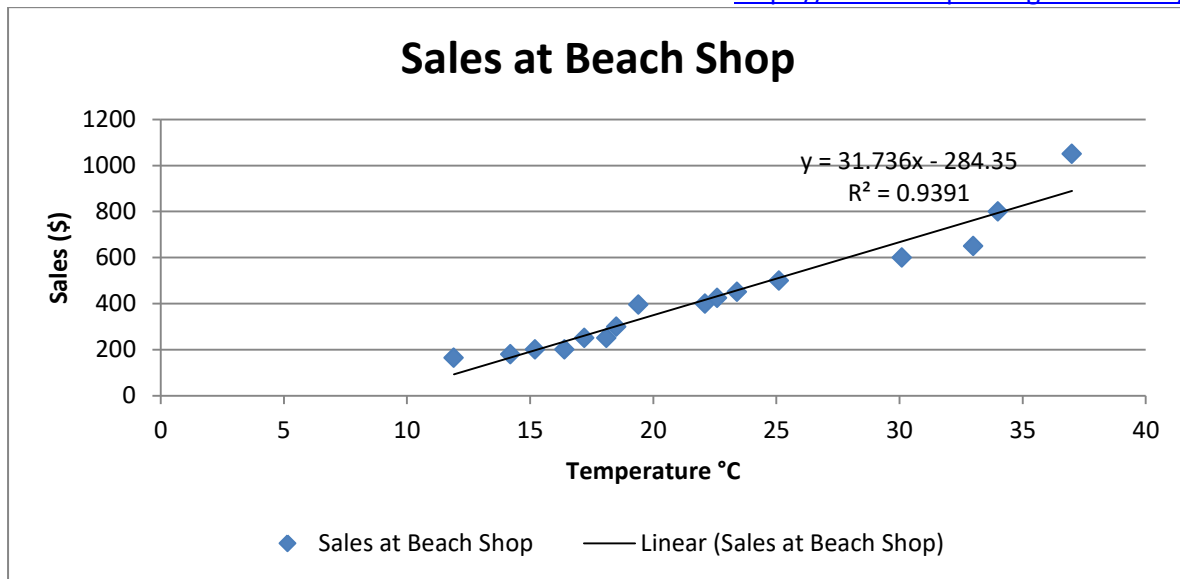
$$a = \frac{(\sum y)(\sum x^2) - (\sum x)(\sum xy)}{n(\sum x^2) - (\sum x)^2}$$

$$b = \frac{n(\sum xy) - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2}$$

$$\text{Thus, } a = [(7835)(8882.06) - (358.2)(196658.6)]/[16(8882.06) - (358.2)^2] = -61.726$$

$$b = [16(196658.6) - (358.2)(7835)]/[16(8882.06) - (358.2)^2] = 24.63$$

Case 2: Sale at the beach shop



The calculation of the regression equation is indicated below.

Temperature (X)	Sales at beach shop (Y)	X ²	Y ²	XY
11.9	165	141.61	27225	1963.5
14.2	180	201.64	32400	2556
15.2	200	231.04	40000	3040
16.4	200	268.96	40000	3280
17.2	251	295.84	63001	4317.2
18.1	251	327.61	63001	4543.1
18.5	300	342.25	90000	5550
19.4	395	376.36	156025	7663
22.1	400	488.41	160000	8840
22.6	425	510.76	180625	9605
23.4	451	547.56	203401	10553.4
25.1	500	630.01	250000	12550
30.1	600	906.01	360000	18060
33	650	1089	422500	21450
34	800	1156	640000	27200
37	1050	1369	1102500	38850

$$\sum X = 358.2, \sum Y = 6818, \sum X^2 = 8882.06, \sum Y^2 = 3830678, \sum XY = 180021.2, n=16$$

The relevant formula for the coefficients of the regression equation is as follows.

$$a = \frac{(\sum y)(\sum x^2) - (\sum x)(\sum xy)}{n(\sum x^2) - (\sum x)^2}$$
$$b = \frac{n(\sum xy) - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2}$$

Thus, $a = [(6818)(8882.06) - (358.2)(180021.2)]/[16(8882.06)-(358.2)^2] = -284.35$

$b = [16(180021.1)-(358.2)(6818)]/[16(8882.06)-(358.2)^2] = 31,736$

Conclusion

It is apparent from the above computations that a linear relationship tends to exist between the temperature and ice cream sales. Clearly, the strength of the relationship in both the cases is found to be strong since the corresponding coefficient of determination is greater than 0.9 which implies that a high amount of variation in ice cream sales is explainable on the basis of the temperature. However, temperature seems to be a more important parameter for sales at the beach than at the mall which is apparent from the higher value of slope for the former which indicates that temperature would have greater effect. This has immense practical implications for the ice cream business which would witness significantly higher sales when the weather is hot.

The underlying technique of linear regression used to define the relationship between the two variables seems appropriate considering the scatter plot which clearly indicates a strong linear trend. This technique is however based on certain assumptions such as presence of linear relationship, absence of multicollinearity and auto correlation along normality of variables. Most of these assumptions seem to have been satisfied here making the given method a superior one in comparison to other alternatives such as high low method and other regression models. In case of non-linearity of relationship, higher power regression or log based regression model could have been considered.

Question 3

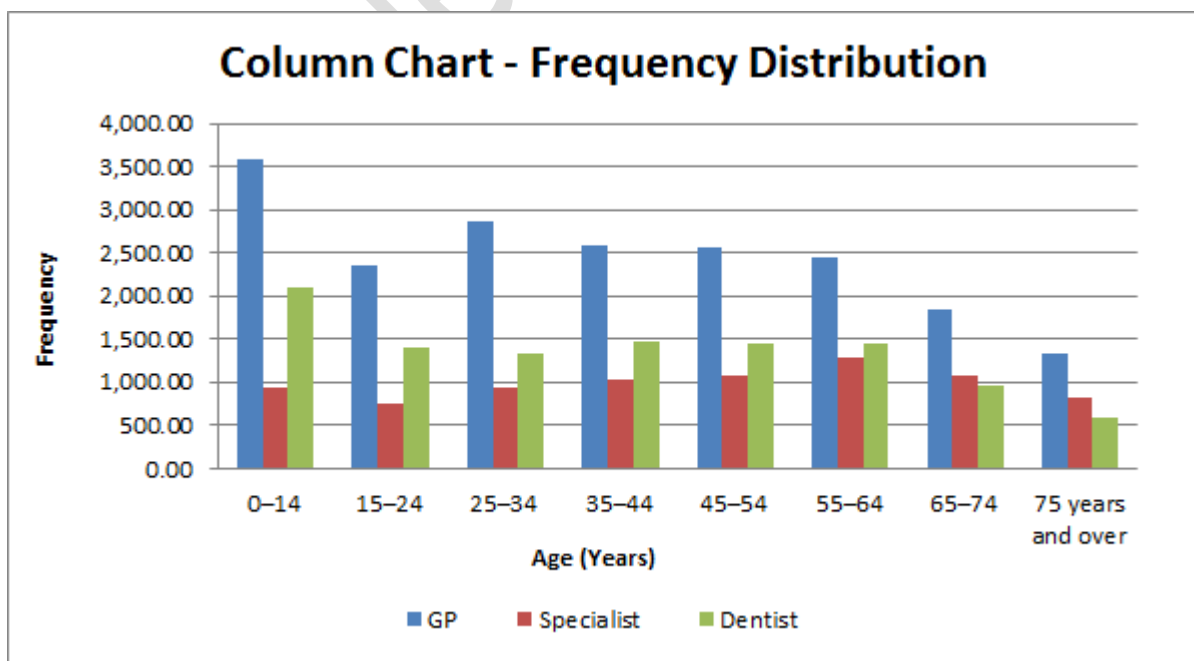
Getting Started

The objective of the given problem is to explore the given data in order to highlight if incremental GP's are required in the region. In order to establish the same, data has been provided which highlights the number of people of various age groups who have availed the services of a GP, specialist and dentist. The methodology used would be descriptive statistics whereby the data would be represented using various graphical aids and also the mean value and dispersion would be determined based on the data. This would then provide an estimation of the average people who tend to attend each of the concerned medical personnel and hence make a strong case for higher GP's to be present in the given region.

Calculations

The highlighted table highlights the number (in '000s of people) who has attended a GP or specialist or dentist in Australia between the period 2014 and 2015.

Age in years	GP	Specialist	Dentist
0-14	3,574.70	927.3	2,101.20
15-24	2,352.50	760.4	1,402.40
25-34	2,860.70	947.7	1,328.20
35-44	2,588.40	1,025.20	1,474.80
45-54	2,572.50	1,076.50	1,444.20
55-64	2,453.40	1,275.30	1,446.10
65-74	1,842.50	1,077.10	971.2
75 years and over	1,322.80	832.3	581



Age in years	GP (f)	Mid point (x)	xf	x- x bar	(x- x bar)^2	(x- x bar)^2 *f
0-14	3574.70	7.00	25022.90	-32.22	1037.90	3710187.12
15-24	2352.50	19.50	45873.75	-19.72	388.74	914510.00
25-34	2860.70	29.50	84390.65	-9.72	94.41	270078.72
35-44	2588.40	39.50	102241.80	0.28	0.08	208.06
45-54	2572.50	49.50	127338.75	10.28	105.75	272043.82
55-64	2453.40	59.50	145977.30	20.28	411.42	1009380.59
65-74	1842.50	69.50	128053.75	30.28	917.09	1689741.09
75 years and over	1322.80	82.00	108469.60	42.78	1830.43	2421292.09
Total	19567.50	356.00	767368.50	42.27	4785.82	10287441.48

$\bar{x} = \frac{\sum xf}{n} = \frac{\sum xf}{\sum f}$	$s = \sqrt{\frac{\sum (x - \bar{x})^2 f}{n-1}}$
Mean	39.22
Standard deviation	22.93

Age in years	Specialist	Mid point (x)	xf	x- x bar	(x- x bar)^2	(x- x bar)^2 *f
0-14	927.30	7.00	6491.10	-32.22	1037.90	962446.22
15-24	760.40	19.50	14827.80	-19.72	388.74	295597.62
25-34	947.70	29.50	27957.15	-9.72	94.41	89472.37
35-44	1025.20	39.50	40495.40	0.28	0.08	82.41
45-54	1076.50	49.50	53286.75	10.28	105.75	113840.69
55-64	1275.30	59.50	75880.35	20.28	411.42	524685.36
65-74	1077.10	69.50	74858.45	30.28	917.09	987799.25
75 years and over	832.30	82.00	68248.60	42.78	1830.43	1523466.44
Total	7921.80	356.00	362045.60	42.27	4785.82	4497390.36

$\bar{x} = \frac{\sum xf}{n} = \frac{\sum xf}{\sum f}$	$s = \sqrt{\frac{\sum (x - \bar{x})^2 f}{n-1}}$
Mean	45.70
Standard deviation	23.83

Age in years	Dentist	Mid point (x)	xf	x- x bar	(x- x bar)^2	(x- x bar)^2 *f
0-14	2101.20	7.00	14708.40	-32.22	1037.90	2180839.00
15-24	1402.40	19.50	27346.80	-19.72	388.74	545168.47
25-34	1328.20	29.50	39181.90	-9.72	94.41	125395.38
35-44	1474.80	39.50	58254.60	0.28	0.08	118.55
45-54	1444.20	49.50	71487.90	10.28	105.75	152725.24
55-64	1446.10	59.50	86042.95	20.28	411.42	594956.09
65-74	971.20	69.50	67498.40	30.28	917.09	890679.26
75 years and over	581.00	82.00	47642.00	42.78	1830.43	1063479.52
Total	10749.10	356.00	412162.95	42.27	4785.82	5553361.51

$\bar{x} = \frac{\sum xf}{n} = \frac{\sum xf}{\sum f}$	$s = \sqrt{\frac{\sum (x - \bar{x})^2 f}{n-1}}$
Mean	38.34
Standard deviation	22.73

Conclusion

Based on the above output, it is apparent that for each age group, the requirement of corresponding GP's is significantly higher in comparison to a specialist and dentist. This is particularly true for children and young people who do not tend to have serious ailments and hence usually require the GP more frequently than others. However, in the middle ages (40-60) the relative importance of specialist tends to increase along with the dentist.

It is apparent that demographics would tend to play a crucial role in establishing the need of more GPs. Hence, the demographics of the concerned region should have a higher population of young population below the age group of 30 years with a comparatively higher concentration of 0-14 years children which are the most frequent visiting patient group for a GP. Also, the additional availability of GP, specialist and dentists in the region should be studied so as to analyse whether the concerned visit data is reflected in the availability pattern.

An alternative means in order to establish the need of GP in the region is to compare the availability of GP in the region per person and compare with the national average. However, in the given process, relevant adjustments would need to be made for the demographic variation from the national average in view of the GP visit pattern observed for the various age groups.